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DIAGNOSING COGNITIVE ERRORS: STATISTICAL PATTERN CLASSIFICATION AND RECOGNITION APPROACH

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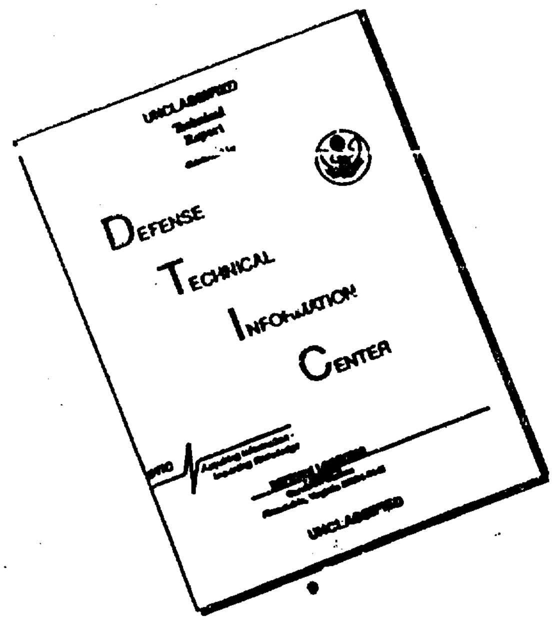

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This paper introduces a probabilistic model that is capable of diagnosing and classifying cognitive errors in a general problem-solving domain. The model is different from the usual deterministic strategies common in the area of artificial intelligence because the item response theory is utilized for handling the variability of response errors. As for illustrating the model, the dataset obtained form a 38-item fraction addition test is used, and the students' responses are classified into 34 groups of misconceptions. These groups are predetermined by the result of an error analysis previously done, and validated with the error diagnostic program written by a typical formal logic approach.

Several deterministic strategies commonly used in the area of Artificial Intelligence (AI) have recently penetrated into the testing of procedural domains and have successfully diagnosed a variety of cognitive errors influencing a student's performance on a test. Such error diagnostic systems (Brown & Burton, 1978; Tatsuoka, Baillie and Yamamoto, 1983; Baillie & Tatsuoka, 1984) are able to provide detailed descriptions of erroneous rules of operation resulting from some sources of misconceptions or incomplete knowledge. Such a diagnosis of student performance on a test provides more valuable and detailed information than the total score of the test can do, and moreover it enables us to optimize remediation of errors and to evaluate instruction and teaching methods.

However, the total numbers of erroneous rules discovered by various cognitive-error-diagnostic systems are quite large: 88 in signed-number addition and subtraction, 70 in fraction addition and 104 in whole number subtraction problems (VanLehn, 1983). A single misconception often produces several different erroneous rules. For example, when adding two fractions with different denominators, many students add the numerators and denominators separately without obtaining the least common denominator of the two numbers. Some students take the larger denominator instead of the common denominator and add the numerators. As listed in Tatsuoka (1984a), there are eleven different erroneous rules which result from a misconception originating with obtaining the least common denominator of two fractions with unlike denominators.

Tatsuoka and Tatsuoka (1982) introduced an index to measure the degree of consistency with which a student applies his/her own rules throughout a test. The index (individual consistency index or ICI) was applied to the domain of signed-number subtraction problems. A summary list of 161 students' ICI values was shown in Tatsuoka and Tatsuoka (1983, pp. 222). The results indicated that quite a few of the students performed irregularly on the parallel forms of subtests. Deterministic strategies cannot handle the variability of response errors efficiently, and hence a stochastic approach is needed.

The purpose of this study is to introduce such a probabilistic model and further discuss some properties of the model (Tatsuoka, 1984b, 1985; Tatsuoka & Baillie, 1982).

Geometric Representation of Binary Response Patterns

The one- or two-parameter logistic model is used in this study. This section begins by introducing a function mapping response binary vectors into a set of ordered pairs. Suppose $\underset{\sim}{x}$ is a vector of scores on n items, $\underset{\sim}{x} = (x_1, x_2, \ldots, x_n)$, and $\underset{\sim}{P}(\theta)$ is a vector of logistic function values whose jth component is given by,

$$P_j(\theta) = \frac{1}{1 + \exp[-1.7a_j(\theta - b_j)]}$$

where b_j is the difficulty of item j, and a_j is the discriminating index. Let $\underset{\sim}{T}(\theta)$ be a vector, $[T(\theta), \ldots, T(\theta)]$ where $T(\theta)$ is the true score, or the average of $P_j(\theta)$ over the n items. Then the cross product of two residuals, $P_j(\theta) - T(\theta)$ and $P_j(\theta) - x_j$ for $j=1,2,\ldots,n$ s a function of $\underset{\sim}{x}$ for a given θ:

$$(1) \quad f_\theta(\underset{\sim}{x}) = (\underset{\sim}{P}(\theta) - \underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)) = \sum_{j=1}^{n} (P_j(\theta) - x_j)(P_j(\theta) - T(\theta)) .$$

The expectation and variance of $f_\theta(\underset{\sim}{x})$ for a fixed θ is given by Equations (2) and (3) (Tatsuoka & Tatsuoka, 1985),

$$(2) \qquad \underset{x_k|\theta}{E} (f_\theta(\underset{\sim}{x}_k)) = 0$$

and

$$(3) \qquad \underset{x_k|\theta}{Var} (f_\theta(\underset{\sim}{x}_k)) = \sum_{j=1}^{n} P_j(\theta)\, Q_j(\theta)(P_j(\theta) - T(\theta))^2 \ .$$

where $\underset{\sim}{x}_k$ is a vector whose total score or weighted sum of a_j is a sufficient statistic of θ in the one- or two-parameter logistic model, respectively. If $f_\theta(\underset{\sim}{x})$ is standardized, it becomes the standardized extended caution index, ζ_2, introduced in Tatsuoka (1984b).

The formula of the standardized extended caution index, ζ_2 is given by Equation (4),

$$(4) \qquad \zeta_2 = \frac{\sum_{j=1}^{n} (P_j(\theta) - x_j)(P_j(\theta) - T(\theta))}{\sqrt{\sum_{j=1}^{n} P_j(\theta) Q_j(\theta)(P_j(\theta) - T(\theta))^2}}$$

Since the total score is a sufficient statistic at the maximum likelihood estimate, MLE $\hat{\theta}$, in the one-parameter logistic model, all response patterns with the same total score have the same MLE $\hat{\theta}$. For example, a ten-item test has 252 different response patterns with the score of five. These patterns correspond to different values of ζ_2 (Tatsuoka, 1985)

The numerator of ζ_2 is divided into two parts in Equation (5); one is a function of $\underset{\sim}{x}$ and the second is a constant value when θ is fixed.

$$(5)\quad (\underset{\sim}{P}(\theta) - \underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)) = -(\underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)) + (\underset{\sim}{P}(\theta), \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta))$$

If a response pattern conforms well to a Guttman scale, the first term of Equation (5) becomes negative. But if the pattern has to be reversed to form a Guttman scale, then $-(\underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta))$ will be positive. The two extreme values of the first term of Equation (5) are obtained by the Guttman and reversed Guttman response patterns, respectively. $f_\theta(\underset{\sim}{x})$ correlates highly with likelihood function, $L = \prod_{j=1}^{n} P_j(\theta)^{x_j} Q_j(\theta)^{1-x_j}$ (Harnisch & Tatsuoka, 1983), and the value of $f_\theta(x)$ reaches the largest or smallest as the likelihood function, L, reaches the largest of smallest, respectively.

If we replace θ by the MXL, $\hat{\theta}$ in Equation (1), then $f_{\hat{\theta}}(\underset{\sim}{x})$ is uncorrelated with $\hat{\theta}$ as well as with the true score $T(\hat{\theta})$ (Tatsuoka, 1985). Therefore, the mapping of $\underset{\sim}{x}$ to the ordered pair of $\hat{\theta}$ and $\zeta_2(\hat{\theta},\zeta_2)$, or to the ordered pair of $T(\hat{\theta})$ and $\zeta_2(T(\hat{\theta}),\zeta_2)$ produce two orthogonal vector spaces, respectively. Tatsuoka (1983, 1985) defined this vector space as "rule space."

The distance between $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ for a given θ in the rule space is given by

$$(6)\qquad \begin{aligned} \text{Dist}\,(\underset{\sim}{x}_1,\underset{\sim}{x}_2) &= |f_\theta(\underset{\sim}{x}_1) - f_\theta(x_2)| \\ &= |\sum_{j=1}^{n} (x_{1j} - x_{2j})(P_j(\theta) - T(\theta))| \quad . \end{aligned}$$

<u>Geometric representation of erroneous rules of operation</u>

The responses to the test items produced by erroneous rules in a procedural domain are scored by a scoring procedure, which may either be component scoring (Tatsuoka, 1983) or the regular right or wrong scoring, and are thus converted into a set of biary response pattern vectors. The component scoring procedure changes the unit of scoring to subcomponents of procedural steps. For example, the answers of signed-number addition and subtraction problems are scored separately for the number and sign parts. By so doing, the regular response patterns are decomposed into two sets of component response patterns. Conversely, the elementwise multiplication of the two component response patterns becomes the regular response pattern. Selection of appropriate components is often the key to representing erroneous rules uniquely by a set of binary component response patterns (Tatsuoka & Tatsuoka, 1981; Tatsuoka, 1983). Similarly, the answers of fraction arithmetic can be decomposed into three subcomponents, the whole number, the numerator and the denominator parts. One of the most typical erroneous rules observed in fraction addition problems is to add the corresponding parts separately. For the problem, 1 4/5 +1 3/7, the answer based on this rule becomes 2 7/12. But for another problem, 4/7 + 2/7, the answer produced by the rule is 6/14, whose numerator happens to coincide with that of the right answer. Therefore, the component response pattern vectors of the two problems will be (1,1) for the whole number part, (0,1) for the numerator part and (0,0) for the denominator part. The elementwise multiplication yields the regular scores, (0,0).

More generally, suppose $\underset{\sim}{x}$, $\underset{\sim}{y}$, and $\underset{\sim}{z}$ are such component response patterns for an erroneous rule, then they will be mapped into the three sets of ordered pairs, $(\hat{\theta}_x, \zeta_{2x})$, $(\hat{\theta}_y, \zeta_{2y})$ and $(\hat{\theta}_z, \zeta_{2z})$ by the mapping function (1). Therefore, in this case, the rule is represented by the points $(\hat{\theta}_x, \zeta_{2x}, \hat{\theta}_y, \zeta_{2y}, \hat{\theta}_z, \zeta_{2z})$ in the six-dimensional space. However, the unit of scoring of the responses to the test items does not have to be a subcomponent of the procedural steps. It could be the regular score of right or wrong, depending on the purpose of analysis. If an analysis is aimed at placing a new student into his/her most appropriate achievement-level group, then the unit of analysis may be the responses to the test items without any finer decomposition into the subprocesses. For example, the items in a mathematics placement test can be decomposed into several subtests measuring geometry, elementary algebra, advanced algebra or trigonometry (Ory, Mayberry & Yamamoto, 1985). Thus, the knowledge measurable from a subtest could be the analysis of interest, and thus replacing an erroneous rule in a procedural domain in the analysis of cognitive errors by several levels of the knowledge in an achievement test.

Illustration of Rule Space with a Fraction Addition Test

All erroneous rules of operation in a procedural domain can be represented as points $(\hat{\theta},\zeta_2)$ in the rule space along with the response patterns obtained from students' performances on a test. Figure 1 shows the rule space constructed for a 38-item fraction addition test given

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Insert Figure 1 about here

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to 595 students in a local junior high school, with four erroneous rules as described in Shaw, et al., (1981). Plus marks (+) represent real data and "o"'s are the four rules listed in Table 1, where the regular scores

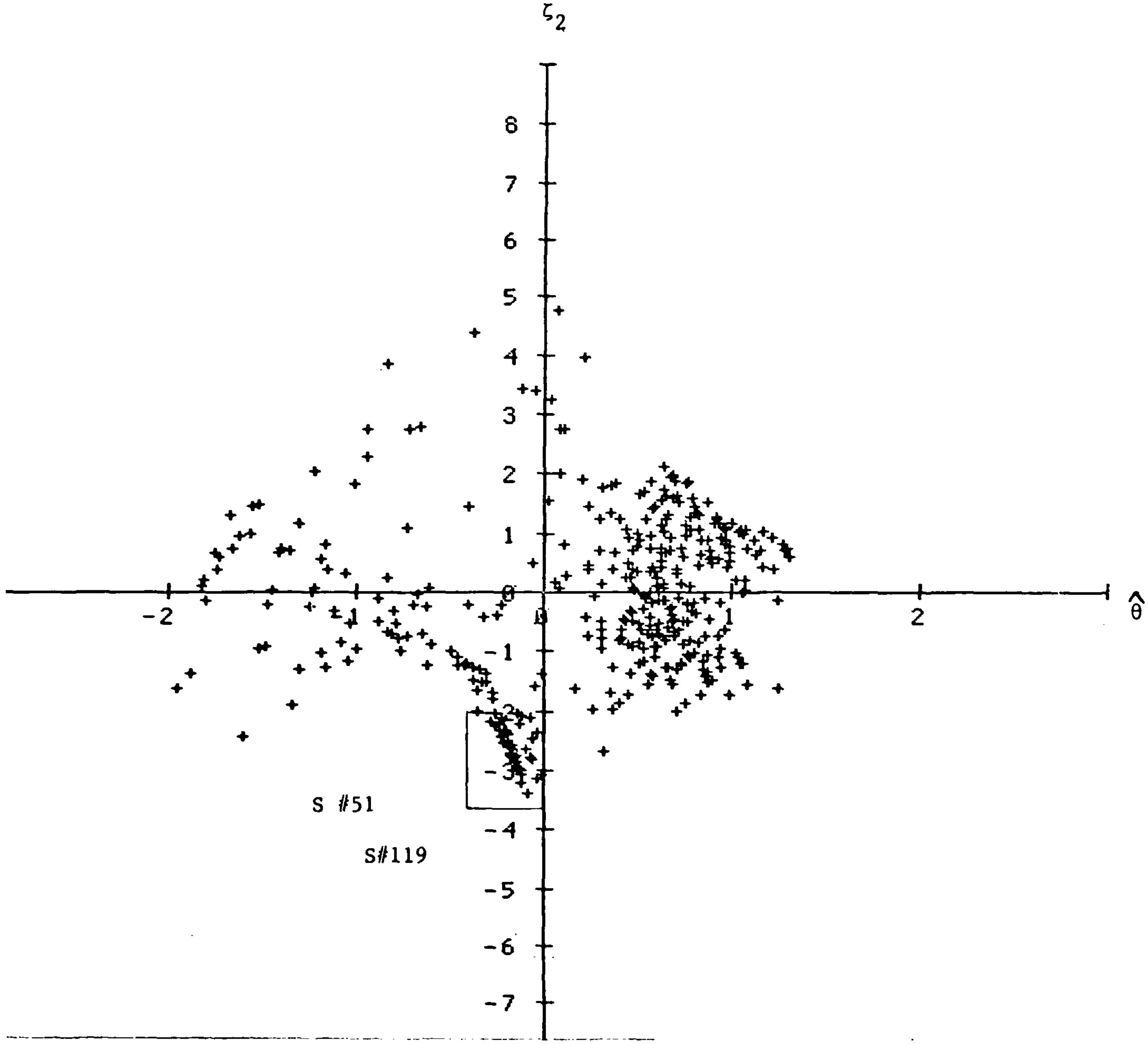

Figure 1: Plot of 595 Students' Scores on a 38-item Fraction Test in the Rule Space

are used to calculate these points. Although only four items are shown in Table 1, the points corresponding to these rules in Figure 1 are obtained

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Insert Table 1 about here

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by using all 38 items in the test. As can be seen in Figure 1, the locations of Rules 13 and 17 are identical, while those of Rules 11 and 10 are farther apart from each other. Since the likelihood of rules correlates highly with the vlues of ζ_2, the rules located close to the θ-axis are observed more often than those located in an upper part of the space, like Rule 10. The response patterns by the regular scoring of Rules 13 and 17 are identical, and hence it is difficult to distinguish between their corresponding points in the rule space. The number of the response patterns similar to those of Rules 13 and 17, ones for the problems of like denominators and zeros for unlike denominators turned out to be large (Tatsuoka, Tatsuoka & Baillie, 1984), and the number of response patterns resulting from use of Rules 13 or 17 with less consistency for the items simplifiable before the addition operation clustered in a box shape, drawn in Figure 1. However, the levels of understanding that produce Rules 13 and 17 are quite different. The source of misconception for Rule 13 originates from obtaining the common denominators of the two like fractions while Rule 17 originates from getting equivalent fractions after the correct denominators are obtained. Therefore, remediaton of the error types relating to Rule 17 should be easier than that of Rule 13. In order to distinguish these different levels of error types, the scores for denominators must be

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levels of courses in mathematics more efficiently and minimizes the number of dropouts and maximizes learning of a new topic for A- or B- students.

However, the new approach requires a couple of technical problems to be solved. A better estimation of ellipses will enhance the analysis results. A better parameter estimation technique will also provide efficient prescriptive information for each student. An accurate decision of classifying each point to one of ellipses will be very important. These techniques are wide open for further research.

possible to evaluate the teacing methods and redesign them for more efficient instruction.

An important characteristic of the model different from other psychometric techniques such as cluster analysis, multidimensional scaling, and factor analysis is that the model can control classification of performances on the test by selecting a particular set of ellipses. These ellipses represent erroneous rules in a procedural domain, or sources of misconceptions producing a variety of erroneous rules of operation, or different achievement levels. Since users of the model controls the input of ellipses according to their need or interest, they can avoid the tricky question about interpretation of factors, or clusters. The quantity expressed by the y-axis represents the typicality of a response pattern with respect to a given sample. Its distance from the x-axis (MLE θ representing state of knowledge) provides the information, to what extent a particular bug is likely happen in the sample. This probablistic interpretation is a unique feature of the rule space model which enhances an already useful prescriptive information about test performances with the likelihood of such an incident.

Since the model is a generalized version of mastery testing or criterion testing, it is applicable to the computer aided instruction on micro-computers as an integrated part of training programs. Diagnosing error types instead of using the total score wll be quite useful in training students. Moreover, the model can be used as an efficient placement tool for administrators of training programs. Ory, et al. (1985) demonstrated that the model places qualified students to various

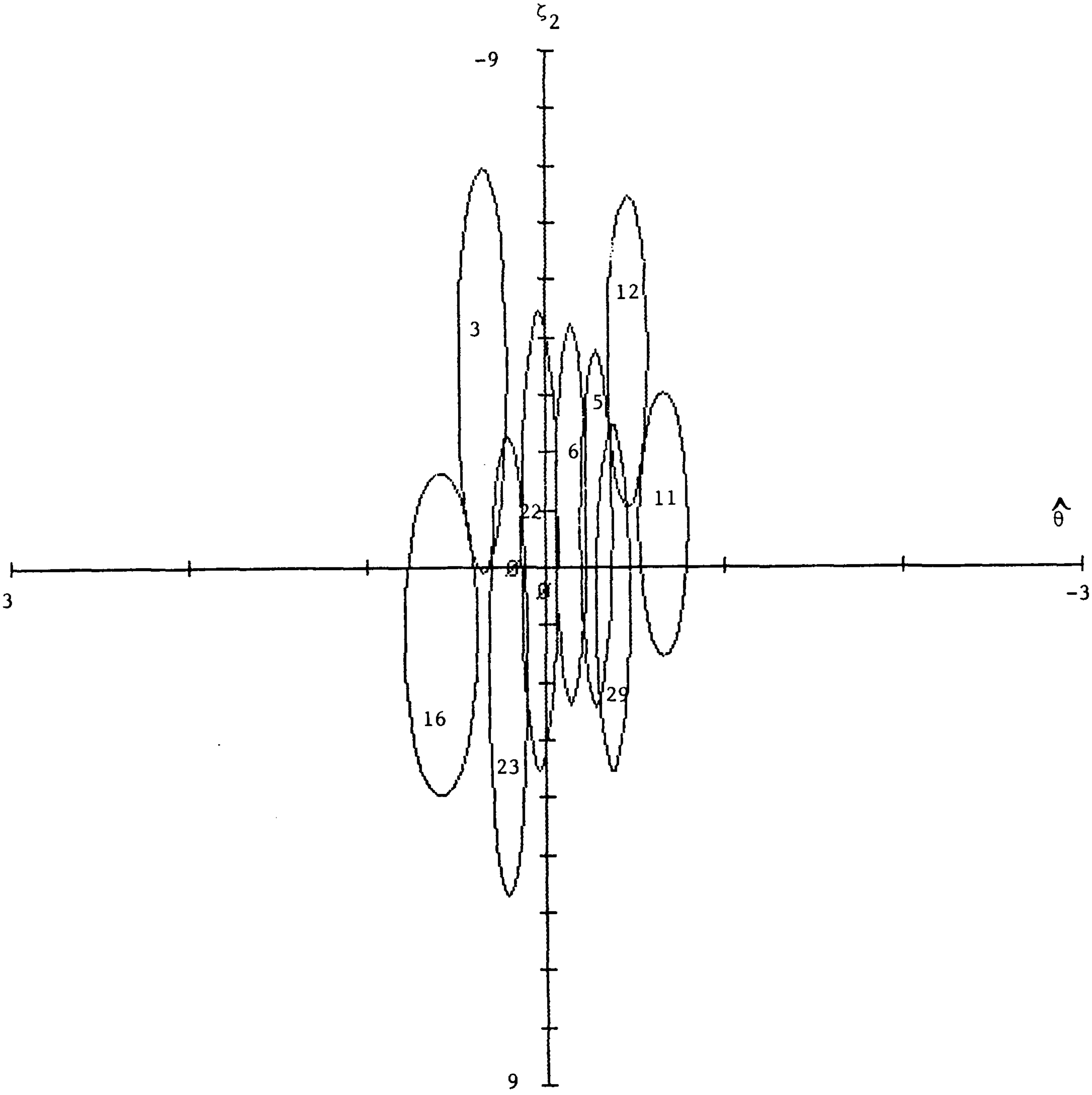

Figure 3: Some Selected Ellipses from Table 3

Table 3

Parameters of Fourteen Ellipses with the Number of Students Classified in Each Ellipse (N = 595)

Set	Frequencies	Centroid ($\hat{\theta}$, ζ_2)
I(1)	4	-0.21, -0.25
II(3)	1	-0.35, 3.41
III(5)	5	0.28, 0.66
IV(6)	37	0.14, 0.91
V(11)	87	0.66, 0.75
VI(12)	12	0.46, 3.74
VII(15)	184	1.93, 0.55
VIII(17)	18	-0.31, -.53
IX(22)	11	-0.04, 0.45
X(23)	23	-0.22, -1.74
XI(28)	2	0.16, -1.00
XII(29)	13	0.38, -0.51
XIII(34)	158	-2.00, 0.75
Unclassified	37	

(continued)

IX(22)	Simplifying error with a combination of addition, equivalent fraction and converting errors.	F+F with NS equivalently, set 5, except for S-type	F+F with S all mixed S and NS
X(23)	Simplifying error with combination of addition equivalent fraction and converting errors.	c=f with NS equivalently set 6 minus S-type	c=f with S all c≠f types S and NS
XI(28)	Simplifying error with combination of other errors	Set 11 minus S-type	Set 11 and all F+F, c=f types with S-types
XII(29)	Simplifying errors	Set 15 minus S-type	Items with S-type
XIII(34)	Mostly adding corresponding parts.	A few items	Almost all items

F+F: Two numbers are fractions such as $\frac{5}{6} + \frac{2}{3}$

Mixed: One or two numbers are mixed such as $1\frac{5}{6} + 2\frac{2}{3}$ or $\frac{5}{6} + 2\frac{2}{3}$

c=f: The denominators are the same in the model $a\frac{b}{c} + d\frac{e}{f}$.

c≠f: The denominators are different in the model $a\frac{b}{c} + d\frac{e}{f}$.

S: The fraction part(s) can be simplified, either by reducing or converting to a mixed number before addition is carried out.

NS: Fraction part(s) are not simplifiable.

*: Set number appeared in Tatsuoka, Tatsuoka & Baillie (1984).

Table 2

Fourteen Sets that Represent Fourteen Different Types of Misconceptions

Set	Incomplete Knowledge or Representative Bug	Item Type Will Get The Right Answer	Item Type Will Get The Wrong Answer
I(1)	Convert mixed numbers to improper fractions by by adding three parts for the numerator and use Rule 13 for answers	F+F and c=f, both S and Ns	Mixed and c=f both S and NS
II(3)	Rule 10	Mixed and c=f, S and NS	all F+F types Mixed and c=f S and NS
III(5)	Convert mixed numbers to wrong improper fractions and use righr rule.	all F+F types S and NS	all mixed numbers, S and NS
IV(6)	If c=f, then use the right rule. If c≠f, then add corresponding parts separately (Rule 13). If c=f, use the right rule. If c≠f, take the least common denominator and add the original numerators (Rule 17).	all c=f types S and NS	all c≠f types S and NS
V(11)	Omit the whole number after using the right procedure on fraction parts.	all F+Ftypes all c=f types S and NS	Mixed and c≠f S and NS
VI(12)		all c=f types all F+F types S and NS	mixed and c=f S and NS
VII(15)	Use the right rule consistently	all items with S and NS	none
VIII(17)	Simplifying error, converting error and addition error	F+F and c=f, NS In other words, Set 1 except for simplifiable items	F+F and c=f S mixed and c≠f both S and NS

(continued)

Insert Tables 2 & 3 about here

Insert Figure 3 about here

The sources of misconception or the incomplete, partial knowledge corresponding to each set are given in the second column, and item types producing the correct answers or wrong answers by the set are listed in the third and fourth columns, respectively. Table 3 summarizes the centroids of the 14 ellipses in the third column and the number of students diagnosed as having the error type associated with each set. Figure 3 shows 14 ellipses which cover the area in the rule space pretty well. By examining which points are found in particular ellipses, 93.8% of students' performances on the 38-item test are diagnosed.

Discussion

A probablistic model that is capable of diagnosing and classifying cognitive errors is introduced in this paper. The model enables us to deal with variability of response errors via item response theory. Since all response patterns are mapped into a two dimensional vector space spanned by MLE θ and ζ_2, which is defined as the rule space, this approach can be used for evaluating different teaching methods or two expert systems constructed by different models of a single problem solving domain. The point is that the two different teaching methods will produce significantly different numbers of students committing different types of bugs (Tatsuoka & Sharabash, 1985), and hence they produce different clusters in the space. By examining these clusters closely, it is

Map representation of misconceptions in the rule space

It is shown that response patterns originating from a single source of misconception or rule with a few slips, cluster in the vicinity of the point representing that rule in the rule space (Tatsuoka & Baillie, 1982). In this section, several sources of misconceptions in fraction addition problems identified by the error analysis of fraction addition (Shaw, *et al.*, 1981; Baillie & Tatsuoka, 1984) will be mapped into various regions of the rule space. Points in these regions follow bivariate normal distributions (Tatsuoka & Tatsuoka, 1985) so that they can be expressed algebraically. As mentioned earlier, the major and minor axes of the ellipses are orthogonal and thus the equations of the ellipses are given by

(7) $$(\underset{\sim}{X} - \underset{\sim}{X}_R)' \, \Sigma^{-1} (\underset{\sim}{X} - \underset{\sim}{X}_R) = \text{const}$$

where $\underset{\sim}{X}$ corresponds to a student's score, $\underset{\sim}{X}_R$ corresponds to the responses to the items generated by Rule R, and the covariance matrix Σ is a diagonal matrix with the variances of θ and ζ_2, respectively. (For more information, refer to Tatsuoka & Tatsuoka, 1985).

Fourteen sets representing different characteristics of the performance on the test are described in Table 2. The classification of these erroneous rules is a tricky task (Johnson, Draper & Soloway, 1982; Tatsuoka, 1984c). Several different approaches to classifying erroneous rules in fraction addition problems are tried, but in this study the method based on a task analysis described in Birenbaum and Shaw (1984) is used to illustrate how our rule space model works.

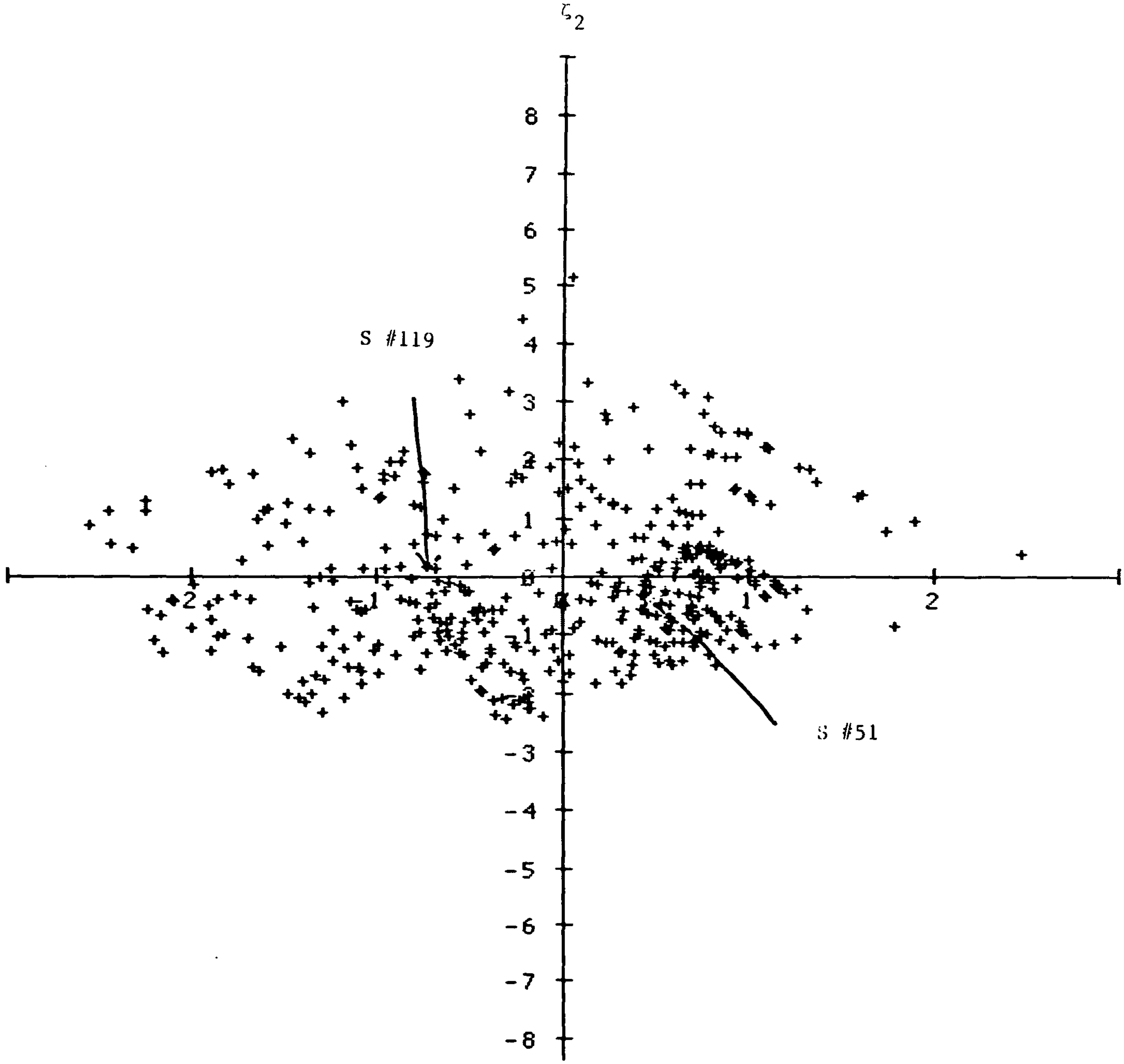

Figure 2: Plot of 595 Students' Performances on Denominators of a 38-Item Fraction Test.

taken into account because the response pattern of each component given in Table 1 shows the denominators produced two different patterns.

Figure 2 is the plot of ordered pairs obtained from the denominator-component scores for the same 38-item fraction addition test used in Figure 1. The cluster bounded by a box in Figure 1 was dispersed in Figure 2, and the two points associated with a consistent application of Rules 13 and 17 resulted in two distant locations as shown in Figure 2.

Insert Figure 2 about here

As for Rule 10, the corresponding point falls in an upper part of the rule space because use of Rule 10 is not popular. Indeed, it is very unusual to see a response pattern consisting of ones for harder problems such as adding two mixed numbers and zeros for the simpler addition of two fractions. The rule space enables us to see two important aspects of pure characteristics attributed to each erroneous rule resulting from sources of misconceptions. The first element, MLE $\hat{\theta}$ represents the state of knowledge where a spcecific error is produced. The second element ζ_2 expresses the extent to which this error type is usual and popular in a given population. Points located closer to the x-axis are usual and frequently observed ones, while points falling in the upper part of the space are unusual and less frequent.

A natural question arises as to whether or not a point that lies very close to Rule 13 is actually yielded by Rule 13 with a few slips (or random errors resulting from no-perfectly systematic application of Rule 13). The next section will address this problem.

Table 1

Four Erroneous Rules of Fraction Addition and Their Responses to Four Items

Item & Answer	Rule 11 Response	R	D	N	W	Rule 13 Response	R	D	N	W
$3\frac{5}{7} + 4\frac{6}{7} = 7\frac{11}{14}$	$7\frac{11}{14}$	0	0	1	1	$7\frac{11}{7}$	1	1	1	1
$= 8\frac{4}{7}$						$= 8\frac{4}{7}$				
$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$	$\frac{2}{5}$	0	0	0	1	$\frac{2}{5}$	0	0	0	1
$\frac{8}{5} + \frac{6}{5} = \frac{14}{5}$	$\frac{14}{10}$	0	1	1	0	$\frac{14}{5}$	1	1	1	1
$= 2\frac{4}{5}$	$= 1\frac{2}{5}$					$= 2\frac{4}{5}$				
$1\frac{1}{2} + \frac{10}{7} = 1\frac{27}{14}$	$2\frac{11}{9}$	0	0	0	1	$2\frac{11}{9}$	0	0	0	1
$= 2\frac{13}{14}$	$= 3\frac{2}{9}$					$= 3\frac{11}{9}$				

Item & Answer	Rule 17 Response	R	D	N	W	Rule 10 Response	R	D	N	W
$3\frac{5}{7} + 4\frac{6}{7} = 7\frac{11}{14}$	$17\frac{11}{7}$	1	1	1	1	$7\frac{11}{7}$	1	1	1	1
$= 8\frac{4}{7}$						$= 8\frac{4}{7}$				
$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$	$\frac{2}{6} = \frac{1}{3}$	0	0	0	1	$2\frac{2}{5}$	0	0	0	0
$\frac{8}{5} + \frac{6}{5} = \frac{14}{5} = 2\frac{4}{5}$	$\frac{14}{5}$	1	1	1	1	$\frac{14}{5}$	0	1	1	0
$= 2\frac{4}{5}$	$= 2\frac{4}{5}$					$= 2\frac{4}{5}$				
$1\frac{1}{2} + \frac{10}{7} = 1\frac{27}{14}$	$2\frac{11}{14}$	0	1	0	0	$2\frac{27}{14}$	1	1	1	0

Rule 11: Adding corresponding parts separately

Rule 13: If the denominators are different, then apply Rule 11; else gets the right answer.

Rule 17: If the denominators are different, then get the common denominator and add the original numerators; else gets the right answer.

Rule 10: Append a one to all fractions and apply Rule 13.

R : The regular score, the anwer is the unit of scoring.
D : The denominator of the answer is the unit of scoring.
N : The numerator of the answer is the unit of scoring.
W : The whole number part of the answer is the unit of scoring.

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